Hydrogen in Electrodynamics. VI
The General Solution

Hans Sallhofer

Forschungs- und Versuchsanstalt der Austria Metall AG, Braunau, Austria

Z. Naturforsch. **45a**, 1361–1366 (1990); received November 12, 1990

After a discussion of the one-component Schrödinger (1926) and the four-component Dirac (1928) representation of hydrogen it is shown that the six-component electrodynamic picture turns out to be considerably simpler and clearer. The computational effort is reduced to a fraction.

1. The Transversality of Hydrogen

Again we start with covariant source-free electrodynamics [1] (22) with its complex components

$$\left.\begin{array}{l}\operatorname{rot} \boldsymbol{E}+\dfrac{\mu}{c}\dot{\boldsymbol{H}}=0\\ \operatorname{rot} \boldsymbol{H}-\dfrac{\varepsilon}{c}\dot{\boldsymbol{E}}=0\\ \operatorname{div} \varepsilon \boldsymbol{E}=0\\ \operatorname{div} \mu \boldsymbol{H}=0\end{array}\right\}_{\operatorname{div}\boldsymbol{E}=\operatorname{div}\boldsymbol{H}=0} \equiv \left.\begin{array}{l}\operatorname{rot} \boldsymbol{E}+\dfrac{\mu}{c}\dot{\boldsymbol{H}}=0\\ \operatorname{rot} \boldsymbol{H}-\dfrac{\varepsilon}{c}\dot{\boldsymbol{E}}=0\end{array}\right\}_{\substack{\operatorname{div}\boldsymbol{E}=\operatorname{div}\boldsymbol{H}=0\\ \boldsymbol{E}\perp\operatorname{grad}\varepsilon\\ \boldsymbol{H}\perp\operatorname{grad}\mu}} \tag{1}$$

where we have ensured the absence of sources by the requirement

$$\operatorname{div} \boldsymbol{E} = \operatorname{div} \boldsymbol{H} = 0\,. \tag{2}$$

From (2) follows

$$\boldsymbol{E}\cdot\operatorname{grad}\varepsilon = 0 \quad \text{and} \quad \boldsymbol{H}\cdot\operatorname{grad}\mu = 0 \tag{3}$$

or

$$\boldsymbol{E}\perp\operatorname{grad}\varepsilon \quad \text{and} \quad \boldsymbol{H}\perp\operatorname{grad}\mu\,. \tag{4}$$

For the case of hydrogen we have to insert the hydrogen-interfraction

$$(\varepsilon_{\text{int}}^{\text{hyd}},\ \mu_{\text{int}}^{\text{hyd}}) = \left(1-\frac{\Phi-m_0c^2}{\hbar\omega},\ 1-\frac{\Phi+m_0c^2}{\hbar\omega}\right) = \left(1-\frac{-e_0^2/r-m_0c^2}{\hbar\omega},\ 1-\frac{-e_0^2/r+m_0c^2}{\hbar\omega}\right) \tag{5}$$

according to [2] (2). grad $\varepsilon_{\text{int}}^{\text{hyd}}$ and $\mu_{\text{int}}^{\text{hyd}}$ then both point in the direction of the radius vector and, according to (4), the field vectors are therefore tangent to spheres around the origin. We call this essential feature, which immediately provides us with a basic and comprehensive picture of the field structure of hydrogen, "central transversality".

2. The Fermion Ansatz

In [4] (21) we have seen that all components of the hydrogen field have to vanish always and everywhere – with nonvanishing energy – because of the half-integer sign ambiguity of angular momentum and spin. This is in best agreement with the experience that the hydrogen atom doesn't reveal itself to the outside as an electromagnetic phenomenon and is thus perceptible only mechanically. For the general solution we therefore require

$$Q_k = 0 \quad (Q = E, H;\ k = 1, \ldots, 3) \tag{6}$$

with

$$U = Q_k \overset{*}{Q}_k \neq 0\,. \tag{7}$$

Furthermore we especially put

$$Q_2^L = \pm i Q_1^L \quad (L = \mathrm{I}, \mathrm{II}), \tag{8}$$

noticing that this requirement is contained in the ansatz (6), as long as both values of the right-hand side of (8) are taken into account simultaneously. – Since only half-integer spin or momentum functions on the right-hand side of (8) can be double-valued, (8) in the end requires solutions with half-integer azimuthal functions. We therefore call (6)–(8) the fermion ansatz of electrodynamics. The purely electrodynamic derivation of the of the Maxwell-Dirac isomorphism from [3] (7) to [3] (20) shows that the solutions of (1) can be represented by means of the Dirac functions Ψ_k^L ($L = \mathrm{I}$,

Reprint requests to Dr. H. Sallhofer, Bahnhofstrasse 36, A-5280 Braunau, Austria.

0932-0784 / 90 / 1100-1361 $ 01.30/0. – Please order a reprint rather than making your own copy.

Dieses Werk wurde im Jahr 2013 vom Verlag Zeitschrift für Naturforschung in Zusammenarbeit mit der Max-Planck-Gesellschaft zur Förderung der Wissenschaften e.V. digitalisiert und unter folgender Lizenz veröffentlicht: Creative Commons Namensnennung-Keine Bearbeitung 3.0 Deutschland Lizenz.

This work has been digitalized and published in 2013 by Verlag Zeitschrift für Naturforschung in cooperation with the Max Planck Society for the Advancement of Science under a Creative Commons Attribution-NoDerivs 3.0 Germany License.

Zum 01.01.2015 ist eine Anpassung der Lizenzbedingungen (Entfall der Creative Commons Lizenzbedingung „Keine Bearbeitung") beabsichtigt, um eine Nachnutzung auch im Rahmen zukünftiger wissenschaftlicher Nutzungsformen zu ermöglichen.

On 01.01.2015 it is planned to change the License Conditions (the removal of the Creative Commons License condition "no derivative works"). This is to allow reuse in the area of future scientific usage.

II; $k=1,\ldots,4$) as follows:

$$\begin{pmatrix} iE_3^{\mathrm{I}} & i(E_1^{\mathrm{II}}-iE_2^{\mathrm{II}}) \\ i(E_1^{\mathrm{I}}+iE_2^{\mathrm{I}}) & -iE_3^{\mathrm{II}} \\ H_3^{\mathrm{I}} & H_1^{\mathrm{II}}-iH_2^{\mathrm{II}} \\ (H_1^{\mathrm{I}}+iH_2^{\mathrm{I}}) & -H_2^{\mathrm{II}} \end{pmatrix} = \begin{pmatrix} \Psi_1^{\mathrm{I}} & \Psi_1^{\mathrm{II}} \\ \Psi_2^{\mathrm{I}} & \Psi_2^{\mathrm{II}} \\ \Psi_3^{\mathrm{I}} & \Psi_2^{\mathrm{II}} \\ \Psi_4^{\mathrm{I}} & \Psi_4^{\mathrm{II}} \end{pmatrix} \tag{9}$$

(Maxwell) (Dirac)

Inserting into the Maxwell side of (9) successively both alternatives of (8) and adding the results we get

$$\begin{pmatrix} iE_3^{\mathrm{I}} & iE_1^{\mathrm{II}} \\ iE_1^{\mathrm{I}} & -iE_3^{\mathrm{II}} \\ H_3^{\mathrm{I}} & H_1^{\mathrm{II}} \\ H_1^{\mathrm{I}} & -H_3^{\mathrm{II}} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \Psi_1^{\mathrm{I}} & \Psi_1^{\mathrm{II}} \\ \Psi_2^{\mathrm{I}} & \Psi_2^{\mathrm{II}} \\ \Psi_3^{\mathrm{I}} & \Psi_3^{\mathrm{II}} \\ \Psi_4^{\mathrm{I}} & \Psi_4^{\mathrm{II}} \end{pmatrix}. \tag{10}$$

This equation relates the electromagnetic field components unambiguously to the Dirac functions, and thus enables us to calculate without much effort the electromagnetic field components according to the scheme

$$Q_k = Q_k^{\mathrm{I}} + Q_k^{\mathrm{II}}, \tag{11}$$

once we know the Dirac functions.

3. The General Hydrogen Solution

Let us now insert into the Dirac side of (10) those Dirac functions which we have obtained in [5] (47)–(50). With

$$\Psi_k^L = \bar{\Psi}_k^L + \overset{+}{\Psi}_k^L \quad (L=\mathrm{I},\mathrm{II};\ k=1,\ldots,4) \tag{12}$$

they are characterized by the D'Alembert structure and, to begin with, permit the following decomposition

$$\begin{pmatrix} iE_3^{\mathrm{I}} & iE_1^{\mathrm{II}} \\ iE_1^{\mathrm{I}} & -iE_3^{\mathrm{II}} \\ H_3^{\mathrm{I}} & H_1^{\mathrm{II}} \\ H_1^{\mathrm{I}} & -H_3^{\mathrm{II}} \end{pmatrix} + \begin{pmatrix} iE_3^{\mathrm{I}} & iE_1^{\mathrm{II}} \\ iE_1^{\mathrm{I}} & -iE_3^{\mathrm{II}} \\ H_3^{\mathrm{I}} & H_1^{\mathrm{II}} \\ H_1^{\mathrm{I}} & -H_3^{\mathrm{II}} \end{pmatrix}$$

$$= \begin{pmatrix} \bar{\Psi}_1^{\mathrm{I}} & \overset{+}{\Psi}_1^{\mathrm{II}} \\ \bar{\Psi}_2^{\mathrm{I}} & \overset{+}{\Psi}_2^{\mathrm{II}} \\ \bar{\Psi}_3^{\mathrm{I}} & \overset{+}{\Psi}_3^{\mathrm{II}} \\ \bar{\Psi}_4^{\mathrm{I}} & \overset{+}{\Psi}_4^{\mathrm{II}} \end{pmatrix} + \begin{pmatrix} \overset{+}{\Psi}_1^{\mathrm{I}} & \bar{\Psi}_1^{\mathrm{II}} \\ \overset{+}{\Psi}_2^{\mathrm{I}} & \bar{\Psi}_2^{\mathrm{II}} \\ \overset{+}{\Psi}_3^{\mathrm{I}} & \bar{\Psi}_3^{\mathrm{II}} \\ \overset{+}{\Psi}_4^{\mathrm{I}} & \bar{\Psi}_4^{\mathrm{II}} \end{pmatrix}. \tag{13}$$

or simply

$$\begin{pmatrix} \bar{\Psi}_1^{\mathrm{I}} & \overset{+}{\Psi}_1^{\mathrm{II}} \\ \bar{\Psi}_2^{\mathrm{I}} & \overset{+}{\Psi}_2^{\mathrm{II}} \\ \bar{\Psi}_3^{\mathrm{I}} & \overset{+}{\Psi}_3^{\mathrm{II}} \\ \bar{\Psi}_4^{\mathrm{I}} & \overset{+}{\Psi}_4^{\mathrm{II}} \end{pmatrix} = \begin{pmatrix} i\check{E}_3^{\mathrm{I}} & i\check{E}_1^{\mathrm{II}} \\ i\check{E}_1^{\mathrm{I}} & -i\check{E}_3^{\mathrm{II}} \\ \check{H}_3^{\mathrm{I}} & \check{H}_1^{\mathrm{II}} \\ \check{H}_1^{\mathrm{I}} & -\check{H}_3^{\mathrm{II}} \end{pmatrix} = \begin{pmatrix} \overset{+}{\Psi}_1^{\mathrm{I}} & \bar{\Psi}_1^{\mathrm{II}} \\ \overset{+}{\Psi}_2^{\mathrm{I}} & \bar{\Psi}_2^{\mathrm{II}} \\ \overset{+}{\Psi}_3^{\mathrm{I}} & \bar{\Psi}_3^{\mathrm{II}} \\ \overset{+}{\Psi}_4^{\mathrm{I}} & \bar{\Psi}_4^{\mathrm{II}} \end{pmatrix}. \tag{14}$$

Continuing, after these preparations, towards the solution of (1) we have to solve the purely electromagnetic equation [3] (19), according to the derivation between [3] (7) and [3] (20), or, taking into account (5), the purely electromagnetic equation [3] (20), that is

$$\left[\gamma\cdot\nabla + \frac{i}{\hbar c}\begin{pmatrix} \left(\hbar\omega + \frac{e_0^2}{r} + m_0c^2\right)\mathbf{1} & 0 \\ 0 & \left(\hbar\omega + \frac{e_0^2}{r} - m_0c^2\right)\mathbf{1} \end{pmatrix}\right]\Psi = 0, \tag{15}$$

which is identical with the Dirac equation of hydrogen. Since the solution of the Dirac equation of hydrogen is known, we may in the following take over the solutions $\Psi_k^{\mathrm{I,II}}$, as well as any needed details of the method of solution from the Dirac theory.

According to [5] (47)–(50) the radial components R_Q of the Dirac functions, taken as solutions of the two pairs of differential equations [5] (31)–(32) and [5] (36)–(37), appear in the two states "I" and "II" as follows

$$\Psi^{\mathrm{I}} = CR_Q^{\mathrm{I}}\, P^{m^{\mathrm{I}}}\, e^{im^{\mathrm{I}}\phi}\, e^{-i\omega^{\mathrm{I}}t} \qquad 0 \leqq |m^{\mathrm{I}}| \leqq l^{\mathrm{I}}, \tag{16}$$

$$\Psi^{\mathrm{II}} = CR_Q^{\mathrm{II}}\, P^{m^{\mathrm{II}}}\, e^{im^{\mathrm{II}}\phi}\, e^{-i\omega^{\mathrm{II}}t} \qquad 0 \leqq |m^{\mathrm{II}}| \leqq l^{\mathrm{II}}. \tag{17}$$

Here the electric radial component R_E, for instance, has the following form for the states I and II:

$$R_E^{\mathrm{I}} = e^{-\lambda r}\sum_{k=0}^{n_r} a_k^{\mathrm{I}}\, r^{q^{\mathrm{I}}+k}, \quad \lambda = \frac{2\pi}{hc}\sqrt{m_0^2\,c^4-(h\nu)^2}, \tag{18}$$

$$q^{\mathrm{I}} = -1+\sqrt{(-l^{\mathrm{I}}-1)^2-\alpha^2}, \quad (l^{\mathrm{I}}=0,1,\ldots), \tag{19}$$

$$a_k^{\mathrm{I}} = \left[\alpha-(q^{\mathrm{I}}+k-l^{\mathrm{I}})\sqrt{\frac{m_0\,c^2-h\,\nu}{m_0\,c^2+h\,\nu}}\right]c_k^{\mathrm{I}}, \tag{20}$$

$$[(q_k^{\mathrm{I}}+k-l^{\mathrm{I}}+1)(q^{\mathrm{I}}+k+l^{\mathrm{I}}+3)+\alpha^2]\,c_{k+1}^{\mathrm{I}} = \frac{4\pi}{hc}[(q^{\mathrm{I}}+k+1)\sqrt{m_0^2\,c^4-(h\nu)^2}-\alpha\,h\,\nu]\,c_k^{\mathrm{I}}, \tag{21}$$

$$\omega^{\mathrm{I}} = \frac{m_0\,c^2}{\hbar\sqrt{1+\dfrac{\alpha^2}{(n_r+\sqrt{(-l^{\mathrm{I}}-1)^2-\alpha^2})^2}}} = \omega(-l^{\mathrm{I}}-1), \tag{22}$$

and

$$R_E^{II}=e^{-\lambda r}\sum_{k=0}^{n_r} a_k^{II}\, r^{q^{II}+k},\quad \lambda=\frac{2\pi}{hc}\sqrt{m_0^2\,c^4-(h\nu)^2}, \tag{23}$$

$$q^{II}=-1+\sqrt{(+l^{II})^2-\alpha^2},\quad (l^{II}=1,2,\ldots), \tag{24}$$

$$a_k^{II}=\left[\alpha-(q^{II}+k+l^{II}+1)\sqrt{\frac{m_0\,c^2-h\nu}{m_0\,c^2+h\nu}}\right]c_k^{II}, \tag{25}$$

$$[(q_k^{II}+k+l^{II}+2)(q^{II}+k-l^{II}+2)+\alpha^2]\,c_{k+1}^{II} = \frac{4\pi}{hc}[(q^{II}+k+1)\sqrt{m_0^2\,c^4-(h\nu)^2}-\alpha\,h\nu]\,c_k^{II}, \tag{26}$$

the different bounds of l^I and l^{II} are to be taken into account in such a way that the magnetic quantum number m is lowered by 1 in the azimuthal function of the state II.

We are now going to compute the general solution $\boldsymbol{E}$ from (11) and (14). In (14) we have, in anticipation, denoted the prospective solution as $\check{\boldsymbol{E}}$, since, as we shall see, it only represents one half of the general solution, – the other half $\hat{\boldsymbol{E}}$ will be obtained by an appropriate substitution. The general solution, therefore, will be of the form

$$E_k=\check{E}_k+\hat{E}_k\,. \tag{32}$$

To begin with, let us turn to the left-hand side of the douple equation (14) and write it out explicitly according to [5] (47)–(50):

$$C\begin{pmatrix} -iR_E^I P_{l^I+1}^{m+1} e^{-i(m+1)\phi-i\omega^I t} & -i(l^{II}+m)\,R_E^{II} P_{l^{II}-1}^{m} e^{i(m-1)\phi-i\omega^{II}t} \\ i(l^I-m+1)\,R_E^I P_{l^I+1}^{m} e^{-im\phi-i\omega^I t} & iR_E^{II} P_{l^{II}-1}^{m+1} e^{im\phi-i\omega^{II}t} \\ R_H^I P_{l^I}^{m+1} e^{-i(m+1)\phi-i\omega^I t} & (l^{II}-m)\,R_H^{II} P_{l^{II}}^{m} e^{i(m-1)\phi-i\omega^{II}t} \\ (l^I+m+1)\,R_H^I P_{l^I}^{m} e^{-im\phi-i\omega^I t} & R_H^{II} P_{l^{II}}^{m+1} e^{im\phi-i\omega^{II}t} \end{pmatrix} = \begin{pmatrix} i\check{E}_3^I & i\check{E}_1^{II} \\ i\check{E}_1^I & -i\check{E}_3^{II} \\ \check{H}_3^I & \check{H}_1^{II} \\ \check{H}_1^I & -\check{H}_3^{II} \end{pmatrix}. \tag{33}$$

$$\omega^{II}=\frac{m_0\,c^2}{\hbar\sqrt{1+\dfrac{\alpha^2}{(n_r+\sqrt{(l^{II})^2-\alpha^2})^2}}}=\omega(l^{II})\,. \tag{27}$$

Obviously the relations

$$R_E^I=R_E(-l^I-1), \tag{28}$$

$$R_E^{II}=R_E(l^{II}) \tag{29}$$

hold good. In (19) and (24), l^I and l^{II} have different bounds. The first of the quantum numbers corresponds to the usual bound as in (16). On the other hand, l^{II} starts with 1, a fact which has been discussed extensively by von Laue [6]. Since m^{II} has to go through zero for the smallest l^{II}-value (=1), it has to be raised in the spherical harmonic of the ansatz (17): (30)

$$\Psi^{II}=CR_Q^{II}\,P^{|m^{II}+1|}\,e^{im^{II}\phi}\,e^{-i\omega^{II}t},\quad 1\leqq|m^{II}+1|\leqq l^{II}.$$

The spherical harmonics should, however, possess the same upper index in the two states I and II in view of their intended separation. We will therefore lower again the upper index of the spherical harmonics in (26), the azimuthal function being taken into account accordingly:

$$\Psi^{II}=CR_Q^{II}\,P^{m^{II}}\,e^{i(m^{II}-1)\phi}\,e^{-i\omega^{II}t}\,. \tag{31}$$

This is possible since we are free to choose the levels of m^I and m^{II} in the scheme (11). On balance, therefore, Here we have lowered the quantum numbers m of Ψ^{II} by one in view of the remarks below (31) to achieve the adjustment of the l-bounds. In addition we have put all constants equal to C.

For $\check{E}_1$ we now get, according to (33) and (11):

$$\check{E}_1=\check{E}_1^I+\check{E}_1^{II} = C\,[(l^I-m+1)\,R_E^I P_{l^I+1}^m e^{-im\phi}e^{-i\omega^I t} -(l^{II}+m)\,R_E^{II} P_{l^{II}-1}^{m} e^{i(m-1)\phi}e^{-i\omega^{II}t}] \tag{34}$$

or, with (28) and (29), as well as (22) and (27),

$$\check{E}_1=C\,[(l^I-m+1)\,R_E(-l^I-1)\,P_{l^I+1}^m e^{-im\phi}e^{-i\omega(-l^I-1)t} -(l^{II}+m)\,R_E(l^{II})\,P_{l^{II}-1}^m e^{i(m-1)\phi}e^{-i\omega(l^{II})t}]\,. \tag{35}$$

We are now trying to separate the two state parameters l^I and l^{II} by means of the averaging double requirement

$$-l^I-1=l^{II}=l=l^I=-l^{II}-1\,. \tag{36}$$

Inserting first the left-hand, then the right-hand requirement of (36) into (35), taking into account the well-known relation for the spherical harmonics

$$P_l^m=P_{-l-1}^m\,, \tag{37}$$

we get the two wave fields

$$-\frac{\check{E}_1}{R_E(l)}=C\,[(l+m)\,P_{l-1}^m e^{-im\phi} +(l+m)\,P_{l-1}^m e^{i(m-1)\phi}]\,e^{-i\omega(l)t}, \tag{38}$$

$$\frac{\check{E}_1}{R_E(-l-1)} = C\,[(l-m+1)\,P^m_{l+1}\,e^{-im\phi} + (l-m+1)\,P^m_{l+1}\,e^{i(m-1)\phi}]\,e^{-i\omega(-l-1)t}, \quad (39)$$

According to (22), or (27) respectively, the wave (38) oscillates with the frequency

$$\omega(l) = \frac{m_0 c^2}{\hbar\sqrt{1+\dfrac{\alpha^2}{(n_r+\sqrt{(l^2-\alpha^2)^2}}}} \quad (40)$$

and the wave (39) of approximately the same amplitude with an almost equal frequency, that is

$$\omega(-l-1) = \frac{m_0 c^2}{\hbar\sqrt{1+\dfrac{\alpha^2}{(n_r+\sqrt{(l+1)^2-\alpha^2)^2}}}} = \omega(l+1), \quad (41)$$

According to well-known theorems of wave theory thus a mean hydrogen frequency

$$\omega = \frac{m_0 c^2}{\hbar\sqrt{1+\dfrac{\alpha^2}{(n_r+\sqrt{(l+1/2)^2-\alpha^2)^2}}}} = \omega(l+1/2), \quad (42)$$

arises. Equation (42) perfectly agrees with the spectroscopically determined hydrogen freqencies [6].

By accepting the mean frequency (42) for (38) and (39), the two fields can be added with the help of [7] (11), and we finally get

$$\check{E}_1 = 4CR^R_E\,P^m_l\cos\theta\,[\pm\cos(m-\tfrac{1}{2})\,\phi]\,(\pm e^{-i\phi/2})\,e^{-i\omega t}, \quad (43)$$

if we introduce

$$R^R_E = \frac{(l+1/2)\,R_E(l)\,R_E(-l-1)}{R_E(l)-R_E(-l-1)} \quad (44)$$

as the "reduced electric radial function". This follows the concept of reduced mass [1] (6) because of the similarity in structure and similar invariance with respect to the interchange ($m^{\mathrm{Re}} \leftrightarrow m^{\mathrm{Im}}$, $l \leftrightarrow -l-1$).

As we have already seen in [4] (21), $\check{E}_1$ has to vanish everywhere and always, in view of the linearity of (1), because of the two azimuthal functions with their sign ambiguities

$$\pm\cos(m-\tfrac{1}{2})\,\phi \quad \text{(momentum function)} \quad \text{and}$$
$$\pm e^{-i\phi/2} \quad \text{(spin function)} \quad (45)$$

in contrast to the energy, which is positive everywhere and always. Keeping this basic fact in mind we are going to write (43), and all other components or partial components, in a simpler form, that is

$$\check{E}_{1-} = 4CR^R_E\,P^m_l\cos\theta\cos(m-\tfrac{1}{2})\,\phi\,e^{-i\phi/2}\,e^{-i\omega t} = 0. \quad (46)$$

Here the sign after the component index denotes which sense of rotation is indicated by the half-integer spin factor.

Because of the vanishing of all components, or partial components respectively, we do not have to pay any attention to their sign and thus simply write them as basically positive quantities.

If we analogously repeat the arguments leading from (33) to (43) for $\check{E}_3$, taking into account the relation [7] (10) instead of [7] (11), we get for this component

$$\check{E}_{3-} = 4CR^R_E\,P^m_l\sin\theta\cos(m+\tfrac{1}{2})\,\phi\,e^{-i\phi/2}\,e^{-i\omega t} = 0. \quad (47)$$

Turning now to the right-hand side of (14), and proceeding as on the left-hand side, we get

$$\check{E}_{1+} = i4CR^R_E\,P^m_l\sin\theta\sin(m+\tfrac{1}{2})\,\phi\,e^{i\phi/2}\,e^{-i\omega t} = 0. \quad (48)$$

$$\check{E}_{3+} = i4CR^R_E\,P^m_l\cos\theta\sin(m-\tfrac{1}{2})\,\phi\,e^{i\phi/2}\,e^{-i\omega t} = 0. \quad (49)$$

In order to obtain the electric partial component $\hat{E}_k$, which together with $\check{E}_k$ should yield the general components according to (32), we interchange the columns in (9), either on the Maxwell or on the Dirac side. This amounts to an interchange of the columns of the central matrix in the double relation (14) – by keeping the Roman index fixed:

$$\begin{pmatrix} \bar{\Psi}^{\mathrm{I}}_1 & \overset{+}{\Psi}{}^{\mathrm{II}}_1 \\ \bar{\Psi}^{\mathrm{I}}_2 & \overset{+}{\Psi}{}^{\mathrm{II}}_2 \\ \bar{\Psi}^{\mathrm{I}}_3 & \overset{+}{\Psi}{}^{\mathrm{II}}_3 \\ \bar{\Psi}^{\mathrm{I}}_4 & \overset{+}{\Psi}{}^{\mathrm{II}}_4 \end{pmatrix} = \begin{pmatrix} i\hat{E}^{\mathrm{I}}_1 & i\hat{E}^{\mathrm{II}}_3 \\ -i\hat{E}^{\mathrm{I}}_3 & i\hat{E}^{\mathrm{II}}_1 \\ \hat{H}^{\mathrm{I}}_1 & \hat{H}^{\mathrm{II}}_3 \\ -\hat{H}^{\mathrm{I}}_3 & \hat{H}^{\mathrm{II}}_1 \end{pmatrix} = \begin{pmatrix} \overset{+}{\Psi}{}^{\mathrm{I}}_1 & \bar{\Psi}^{\mathrm{II}}_1 \\ \overset{+}{\Psi}{}^{\mathrm{I}}_2 & \bar{\Psi}^{\mathrm{II}}_2 \\ \overset{+}{\Psi}{}^{\mathrm{I}}_3 & \bar{\Psi}^{\mathrm{II}}_3 \\ \overset{+}{\Psi}{}^{\mathrm{I}}_4 & \bar{\Psi}^{\mathrm{II}}_4 \end{pmatrix}. \quad (50)$$

As an be immediately seen, we get as a by-product of this interchange of columns an interchange of components of the form

$$\hat{E}_1 \leftrightarrow \hat{E}_3 \quad \text{and} \quad \hat{H}_1 \leftrightarrow \hat{H}_3, \quad (51)$$

which has to be taken into account below.

Writing out explicitly the central matrix of the double relation (50) and then, exactly as before with (14), turning successively to the two sides, the respective electric partial components – taking into account (51) – are obtained as:

$$\hat{E}_{1-} = i4CR^R_E\,P^m_l\cos\theta\sin(m-\tfrac{1}{2})\,\phi\,e^{-i\phi/2}\,e^{-i\omega t} = 0. \quad (52)$$

$$\hat{E}_{3-}=i4CR_E^R\,P_l^m\sin\theta\sin(m+\tfrac{1}{2})\phi\,e^{-i\phi/2}\,e^{-i\omega t}=0. \quad (53)$$

$$\hat{E}_{1+}=4CR_E^R\,P_l^m\sin\theta\cos(m+\tfrac{1}{2})\phi\,e^{i\phi/2}\,e^{-i\omega t}=0. \quad (54)$$

$$\hat{E}_{3+}=4CR_E^R\,P_l^m\cos\theta\cos(m-\tfrac{1}{2})\phi\,e^{i\phi/2}\,e^{-i\omega t}=0. \quad (55)$$

By finally putting together the partial components of equal spin from (46)–(49) and (52)–(55) according to (32), and by taking into account (8) and (11) for the 2-components, we get for the left-spinning hydrogen field

$$E_-^{\text{hyd}}=\begin{cases}E_{1-}=4CR_E^R\,P_l^{|m|}\cos\theta\,e^{i(m-1/2)\phi}\,e^{i(-\phi/2-\omega t)}=0, & (56)\\ E_{2-}=i4CR_E^R\,P_l^{|m|}\cos\theta\,e^{i(m-1/2)\phi}\,e^{i(-\phi/2-\omega t)}=0, & (57)\\ E_{3-}=4CR_E^R\,P_l^{|m|}\sin\theta\,e^{i(m+1/2)\phi}\,e^{i(-\phi/2-\omega t)}=0, & (58)\end{cases}$$

and an analogous expression for the right-spinning one. Putting together both spin states, we can write

$$E_\pm^{\text{hyd}}=\begin{pmatrix}4CR_E^R\,P_l^{|m|}\cos\theta\,e^{i(m-1/2)\phi}\,e^{i(\pm\phi/2-\omega t)}\\ i\,4CR_E^R\,P_l^{|m|}\cos\theta\,e^{i(m-1/2)\phi}\,e^{i(\pm\phi/2-\omega t)}\\ 4CR_E^R\,P_l^{|m|}\sin\theta\,e^{i(m+1/2)\phi}\,e^{i(\pm\phi/2-\omega t)}\end{pmatrix}=0 \quad (59)$$

The magnetic field components we obtain in the same manner as the electric ones by first inserting the relations [5] (44), (45)

$$l_H^{\text{I}}=l_E^{\text{I}}-1 \quad \text{and} \quad l_H^{\text{II}}=l_E^{\text{II}}+1\,. \quad (60)$$

They read

$$H_\pm^{\text{hyd}}=\begin{pmatrix}4CR_H^R\,P_l^{|m|}\cos\theta\,e^{i(m-1/2)\phi}\,e^{i(\pm\phi/2-\omega t)}\\ i\,4CR_H^R\,P_l^{|m|}\cos\theta\,e^{i(m-1/2)\phi}\,e^{i(\pm\phi/2-\omega t)}\\ 4CR_H^R\,P_l^{|m|}\sin\theta\,e^{i(m+1/2)\phi}\,e^{i(\pm\phi/2-\omega t)}\end{pmatrix}=0 \quad (61)$$

Obviously we are back to Schrödinger again. In essence only the spin function has been added. This represents an improvement which had to appear imperatively, since electrodynamics as a theory with clear concepts necessarily has to bring about a conceptually clear description of spin. The Dirac theory, in comparison, is not able to explicitly express spin in the spinor components of its hydrogen field since it cannot accomplish the spin generating interchange of components achieved by electrodynamics in its equation (10). The Maxwell theory shows spin to be a pure phase phenomenon, i.e., the spin wave revolves, in the coordinate system chosen here, around the 3-axis as a pure phase wave, slowly near the center, far away in the far-field with arbitrary speed. A deeper insight into the perculiarities of the electrodynamic spin wave fields of half-integer spin particles requires profound analyses. –Especially the exciting idea of a substantiated spin field which rotatingly transports not the phases but the field strengths themselves reminds of Feinberg's tachyons, and still is far from a conceivable credibility and from the almost unforeseeable applications in a future interstellar communication technology.

The most essential improvement of the spin and momentum representation obviously lies in the cancelling action of the azimuthal functions which adjusts the field to the empirical electrodynamic imperceptibility of hydrogen: electrodynamic particle fields with half-integer spin are represented in any point of configuration space as a standing wave with a cancelling counter-wave, so that the particle field vanishes everywhere and always, but everywhere constitutes a positive source of energy, constant in time.

A first preliminary but basic picture of electrodynamic hydrogen is offered by the central transversality (4)–(5). It depicts the hydrogen field as a light wave impinging from the outside – from all directions – on a point, being reflected by itself at that point and running outward again. The incoming and outgoing waves form a standing wave field with a high concentration of energy around the point of reflection, the center of the hydrogen field. The standing concentration of energy at the center is what we perceive as the hydrogen atom. – This picture removes in an unsurpassably simple manner the contradiction of the century of contemporary physics, the wave-particle-dilemma.

On closer inspection it becomes clear that the incoming and outgoing light wave breaks up into a field twin. As it already becomes evident from Ehrenfest's theorem [1] (5)–(17), two light fields of equal amplitude take part in the energy localization at a point, which influence each other, i.e., which mutually refract one another. In our method of solution they are taken into account in such a way that one light field is carried through the calculation as real, the other one as imaginary. In other words: The two centrally transverse twin fields of hydrogen demand calculation with complex components. – Ehrenfest's theorem, in a one-component calculation, shows that the centers of energy of the two twin fields obey Newton's mechanics. The cause of this mechanical interaction are the two twins of that Kepler system which reflects itself in the hydrogen spectrum and lets us perceive the hydrogen atom mechanically.

Finally subjecting the general solution (59), which is still open with respect to the signs of the components according to the statements made after (46), to the

central transversality condition (4)–(5), the following must be valid:

$$\boldsymbol{E}\cdot\operatorname{grad}\varepsilon_{\mathrm{int}}^{\mathrm{hyd}}=\boldsymbol{E}\cdot\boldsymbol{r}^{0}=e^{i(m+1/2)\phi}\;e^{i(\pm\phi/2-\omega t)}$$
$$\cdot\begin{pmatrix}4CR_E^R\;P_l|^m|\cos\theta\,e^{-i\phi}\\ i4CR_E^R\;P_l|^m|\cos\theta\,e^{-i\phi}\\ 4CR_E^R\;P_l|^m|\sin\theta\end{pmatrix}\cdot\begin{pmatrix}\sin\theta\cos\phi\\ \sin\theta\sin\phi\\ \cos\theta\end{pmatrix}=0. \tag{62}$$

The condition (62), first of all, is fulfilled trivially because of the cancelling functions of the vector (59). Beyond that it is fulfilled if we omit the cancelling functions and attribute to the first two components a positive and to the third one a negative sign.

For the electromagnetic energy we get from (59) and (61)

$$U^{\mathrm{hyd}}=\boldsymbol{E}_{\pm}^{\mathrm{hyd}}\cdot\overset{*}{\boldsymbol{E}}_{\pm}^{\mathrm{hyd}}+\boldsymbol{H}_{\pm}^{\mathrm{hyd}}\overset{*}{\boldsymbol{H}}_{\pm}^{\mathrm{hyd}}\neq 0\,, \tag{63}$$

where the asterisk denotes complex conjugation, as should be expected.

[1] H. Sallhofer, Z. Naturforsch. **43a,** 1039 (1988).
[2] H. Sallhofer, Z. Naturforsch. **44a,** 504 (1989).
[3] H. Sallhofer, Z. Naturforsch. **41a,** 468 (1986); and corrigenda in H. Sallhofer, Z. Naturforsch. **41a,** 1087 (1986).
[4] H. Sallhofer, Z. Naturforsch. **44a,** 715 (1989).
[5] H. Sallhofer, Z. Naturforsch. **45a,** 1038 (1990).
[6] M. v. Laue, Handbuch der Radiologie, Bd. VI, 1, pp. 102 ff., Leipzig 1933.
[7] H. Sallhofer, Z. Naturforsch. **44a,** 167 (1989).